DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

Test Booklet No. : 28501

Series A

TEST BOOKLET

Paper—II

Part–II (ACCOUNTANCY/STATISTICS/MATHEMATICS)

(Objective Type)

Time Allowed : 2 Hours **Full Marks : 100**

Read the following instructions carefully before you begin to answer the questions :

1. The name of the Subject, Roll Number as mentioned in the Admission Certificate, Test Booklet No. and Series are to be written legibly and correctly in the space provided on the Answer-Sheet with Black/Blue ballpoint pen.
2. **Answer-Sheet without marking Series as mentioned above in the space provided for in the Answer-Sheet shall not be evaluated.**
3. All questions carry equal marks.

   The Answer-Sheet should be submitted to the Invigilator.

   Directions for giving the answers : Directions for answering questions have already been issued to the respective candidates in the 'Instructions for marking in the OMR Answer-Sheet' along with the Admit Card and Specimen Copy of the OMR Answer-Sheet.

   Example :

   Suppose the following question is asked :

   The capital of Bangladesh is
   (A) Chennai
   (B) London
   (C) Dhaka
   (D) Dhubri

   You will have four alternatives in the Answer-Sheet for your response corresponding to each question of the Test Booklet as below :

   Ⓐ Ⓑ Ⓒ Ⓓ

   In the above illustration, if your chosen response is alternative (C), i.e., Dhaka, then the same should be marked on the Answer-Sheet by blackening the relevant circle with a Black/Blue ballpoint pen only as below :

   Ⓐ Ⓑ ● Ⓓ

   The example shown above is the only correct method of answering.
4. Use of eraser, blade, chemical whitener fluid to rectify any response is prohibited.
5. Please ensure that the Test Booklet has the required number of pages (56) immediately after opening the Booklet. **Students can attend questions of any one subject—Accountancy or Statistics or Mathematics.** In case of any discrepancy, please report the same to the Invigilator.
6. No candidate shall be admitted to the Examination Hall/Room 20 minutes after the commencement of the examination.
7. No candidate shall leave the Examination Hall/Room without prior permission of the Supervisor/ Invigilator. No candidate shall be permitted to hand over his/her Answer-Sheet and leave the Examination Hall/Room before expiry of the full time allotted for each paper.
8. No Mobile Phone, Electronic Communication Device, etc., are allowed to be carried inside the Examination Hall/Room by the candidates. Any Mobile Phone, Electronic Communication Device, etc., found in possession of the candidate inside the Examination Hall/Room, even if on off mode, shall be liable for confiscation.
9. No candidate shall have in his/her possession inside the Examination Hall/Room any book, notebook or loose paper, except his/her Admission Certificate and other connected papers permitted by the Commission.
10. Complete silence must be observed in the Examination Hall/Room. No candidate shall copy from the paper of any other candidate, or permit his/her own paper to be copied, or give, or attempt to give, or obtain, or attempt to obtain irregular assistance of any kind.
11. This Test Booklet can be carried with you after answering the questions in the prescribed Answer-Sheet.
12. Noncompliance with any of the above instructions will render a candidate liable to penalty as may be deemed fit.
13. No rough work is to be done on the OMR Answer-Sheet. You can do the rough work on the space provided in the Test Booklet.

N.B. : There will be negative marking @ 0·25 per 1 (one) mark against each wrong answer.

SEAL

 [No. of Questions : 100

OR

MATHEMATICS

1. Let $|z| \geq \text{Re}(z)$ for a complex number $z = x + iy$. The equality occurs if

(A) z is real and $x \geq 0$

(B) z is real and $x < 0$

(C) z is imaginary and $x > 0$

(D) None of the above

2. If the ratio of $\frac{z-i}{z-1}$ is purely imaginary, where z is any complex number, then z lies on a circle whose centre is the point

(A) $\frac{1}{2} + i\frac{1}{\sqrt{2}}$

(B) $\frac{1}{2} - i\frac{1}{\sqrt{2}}$

(C) $\frac{1}{2} + i\frac{1}{2}$

(D) $\frac{1}{2} - i\frac{1}{4}$

3. The roots of the equation $x^3 - 3x^2 + kx + 3 = 0$ are in AP. Then the value of k is

(A) 1

(B) 2

(C) − 2

(D) − 1

4. Let A be a matrix, where

$$A = \begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$$

Find the rank of the matrix A.

(A) 1

(B) 2

(C) 3

(D) 4

5. If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

(A) I

(B) A

(C) $I - A$

(D) $3A$

6. Let A be a square matrix of order 3×3. Then $|kA|$ is equal to

(A) $k|A|$

(B) $k^2|A|$

(C) $k^3|A|$

(D) $2|A|$

7. If P, Q, R, S are (1, 2, 5), (− 2, 1, 3), (4, 4, 2), (2, 1, − 4) respectively, then the projection of PQ on RS is

(A) 2/7

(B) $\sqrt{5}/3$

(C) 3

(D) 27

8. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$. Then $\vec{a} \times \vec{b}$ is a unit vector if the angle between $\vec{a}$ and $\vec{b}$ is

(A) $\pi/6$

(B) $\pi/4$

(C) $\pi/3$

(D) $\pi/2$

9. If the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$ and $3\hat{i} + a\hat{j} + 5\hat{k}$ are coplanar, then the value of the constant a is

(A) -2

(B) 3

(C) -4

(D) 4

10. The angle between the lines $x^2 + xy - 6y^2 = 0$ is

(A) 75°

(B) 120°

(C) 125°

(D) 135°

11. The least positive integral value of n, for which

$$\begin{bmatrix} \cos \pi/4 & \sin \pi/4 \\ -\sin \pi/4 & \cos \pi/4 \end{bmatrix}^n$$

is an identity matrix, is

(A) 4

(B) 8

(C) 12

(D) 16

12. The eigenvalues of the matrix

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

are

(A) 0, 3, 15

(B) 0, 2, 17

(C) -1, 3, 9

(D) -1, -3, 12

13. The value of i^i is

(A) $e^{-2n\pi}$

(B) $e^{-(4n+1)\pi}$

(C) $e^{-(4n-1)\pi/8}$

(D) $e^{-(4n+1)\pi/2}$

14. The value of $(1+i)^6 + (1-i)^6$ is

(A) $2i - 6$

(B) $2^{-2} \cos(\pi/4)$

(C) $8 \cos(2\pi/3)$

(D) $16 \cos(3\pi/2)$

15. If a and b are real numbers, then the value of

$$\sin\left\{i \log\left(\frac{a - ib}{a + ib}\right)\right\}$$

is

(A) $ab/(a^2 + b^2)$

(B) $2ab/(a^2 + b^2)$

(C) $ab/(a^2 - b^2)$

(D) $2ab/(a^2 - b^2)$

16. If α is a multiple root of order 3 of the equation $x^4 + bx^2 + cx + d = 0$, then the value of α is

(A) 0

(B) $4b/(27c)$

(C) $-8d/(3c)$

(D) $8d/(9b)$

17. If $1+i$ is a root of the equation $x^4 + x^2 - 2x + 6 = 0$, then find the other roots.

(A) $1-i,\ -1 \pm i\sqrt{2}$

(B) $1-i,\ 1 \pm i\sqrt{2}$

(C) $1-i,\ \pm i\sqrt{2}$

(D) None of the above

18. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + p = 0$, where $q \neq 1$, then the value of $\tan^{-1}\alpha + \tan^{-1}\beta + \tan^{-1}\gamma$ in radian is

(A) 0

(B) $\pi/4$

(C) $3\pi/2$

(D) $n\pi$

19. The congruence $x + 50 \equiv 39 \pmod{7}$ possesses

(A) one solution

(B) two solutions

(C) many solutions

(D) no solution

20. If p is prime, then

(A) $\underline{|p-1} + p \equiv 0 \pmod{p}$

(B) $\underline{|p-1} + 1 \equiv 0 \pmod{p}$

(C) $\underline{|p+1} + 1 \equiv 0 \pmod{p}$

(D) $\underline{|p} + 1 \equiv 0 \pmod{p}$

21. The number of complex roots of the equation $x^6 + x^4 + x^2 + x + 3 = 0$ is

(A) 2

(B) 4

(C) either 2 or 0

(D) either 4 or 6

22. The rank of the matrix

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

is

(A) 1

(B) 2

(C) 3

(D) 4

23. For what value of the parameter λ will the following equations fail to have unique solution?

$$3x - y + \lambda z = 1$$
$$2x + y + z = 2$$
$$x + 2y - \lambda z = -1$$

(A) $\lambda = -7/2$

(B) $\lambda = -6$

(C) $\lambda = -2/3$

(D) None of the above

24. If α is the characteristic root of a non-singular matrix A, then one of the characteristic roots of adj A is

(A) $1/\alpha$

(B) $\alpha/|A|$

(C) $|A|/\alpha$

(D) $\alpha|A|$

25. If A and B are 3×3 real matrices such that rank $(AB)=1$, then the rank (BA) cannot be

(A) 0

(B) 1

(C) 2

(D) 3

26. Let A be a 3×3 matrix with eigenvalues -1, 1, 3. Then

(A) A^2+A is non-singular

(B) A^2-A is non-singular

(C) A^2+3A is non-singular

(D) A^2-3A is non-singular

27. The characteristic polynomial for the matrix

$$A=\begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ -2 & -2 & 2 & 0 \\ 1 & 1 & -1 & 0 \end{bmatrix}$$

is

(A) $x^2(x+1)^2$

(B) $(x-1)^2(x+1)^2$

(C) $x^2(x-1)^2$

(D) None of the above

28. The distance between the planes $2x+3y+5z=4$ and $4x+6y+8z=12$ is

(A) $\frac{2}{\sqrt{29}}$ unit

(B) $\frac{4}{\sqrt{26}}$ unit

(C) 3 units

(D) $\frac{3}{\sqrt{31}}$ unit

29. The unit vector parallel to $\vec{a}=3\hat{i}+4\hat{j}-2\hat{k}$ is

(A) $\sqrt{29}(3\hat{i}+4\hat{j}-2\hat{k})$

(B) $\frac{1}{\sqrt{29}}(3\hat{i}+4\hat{j}-2\hat{k})$

(C) $29(3\hat{i}+4\hat{j}-2\hat{k})$

(D) $\frac{1}{29}(3\hat{i}+4\hat{j}-2\hat{k})$

30. $\lim_{n\to\infty}\frac{1+\sqrt{2}+\sqrt[3]{3}+\sqrt[4]{4}+\ldots+\sqrt[n]{n}}{n}=?$

(A) 1

(B) 0

(C) ∞

(D) n^{-n+1}

31. The value of $\lim_{n \to \infty} \frac{\cos n\pi}{n}$ is

(A) -1

(B) 1

(C) 0

(D) π

32. The sequence $\{a_n\}$, defined by $a_n = \sqrt{n+1} - \sqrt{n}\ \forall n \in \mathbb{N}$, converges to

(A) 1

(B) 2^{-1}

(C) 0

(D) $2^{-3/2}$

33. The sequence $\{a_n\}$, where

$$a_n = 1 + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \dots + \frac{1}{\lfloor n}$$

is

(A) convergent and $2 \le \lim_{n \to \infty} a_n \le 3$

(B) convergent and $2 < \lim_{n \to \infty} a_n < 3$

(C) divergent

(D) convergent and $4 \le \lim_{n \to \infty} a_n < 7$

34. The series

$$\sum_{n=2}^{\infty} \frac{1}{n^2 \log n}$$

(A) is bounded above

(B) converges

(C) diverges

(D) None of the above

35. Suppose that A and B are two non-empty subsets of $\mathbb{R}$ that satisfy the property $a \le b$ for all $a \in A$ and all $b \in B$. Then

(A) $\sup A \ge \inf B$

(B) $\sup A \le \inf B$

(C) $\sup A > \inf B$

(D) $\sup A < \inf B$

36. If $f : \mathbb{R}^2 \to \mathbb{R}$ is defined by

$$f(x, y) = \begin{cases} x^3 / (x^2 + x^4), & \text{if } (x, y) \ne (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$$

then

(A) $f_x(0, 0) = 0$ and $f_y(0, 0) = 0$

(B) $f_x(0, 0) = 0$ and $f_y(0, 0) = 1$

(C) $f_x(0, 0) = 1$ and $f_y(0, 0) = 0$

(D) $f_x(0, 0) = 1$ and $f_y(0, 0) = 1$

37. If $[x]$ denotes the greatest integer not greater than x, find the value of

$$\int_0^3 [x]\, dx$$

(A) 9/4

(B) 9

(C) 1/3

(D) 3

38. The derived set of the set of irrational numbers is

(A) the set of real numbers

(B) the set of rational numbers

(C) the set of irrational numbers

(D) the set of numbers of the form $\frac{1}{2^n}$, where $n \in \mathbb{Z}$

39. The set

$$S = \left\{1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{n}, \ldots\right\}, \ n \in \mathbb{N}$$

is

(A) dense in $\mathbb{R}$

(B) dense in itself

(C) not dense in itself

(D) a closed set

40. Every compact subset of $\mathbb{R}$ is

(A) bounded and closed

(B) bounded but not closed

(C) closed but not bounded

(D) neither bounded nor closed

41. The integrating factor of the differential equation

$$\frac{dy}{dx} + \frac{2}{x} y = \frac{y^3}{x^3}$$

is

(A) x^4

(B) x^{-4}

(C) $x^{3/2}$

(D) x^{-1}

42. The integrating factor of the differential equation

$$(2xy + 3x^2 y + 6y^2)dx + (x^2 + 6y^2)dy = 0$$

is

(A) x^3

(B) y^3

(C) e^{3x}

(D) e^{3y}

43. If $y = (x^2 - 1)^n$, then the value of y_{2n} is

(A) $\lfloor n$

(B) $2 \lfloor n$

(C) $\lfloor 2n$

(D) $2 \lfloor n-1$

44. The particular integral of the differential equation

$$(D^3 - 2D^2 - 5D + 6)y = e^{3x}$$

is

(A) $x \cdot e^{-3x}/10$

(B) $x \cdot e^{3x}/10$

(C) $x/10$

(D) $e^{-3x}/10$

45. The complementary function of the differential equation $(D^2-4)y = x^2$ is

(A) $c_1e^{2x} + c_2e^{-2x} + x^2$

(B) $c_1e^{2x} + c_2e^{-2x} - x^2$

(C) $c_1e^{2x} + c_2e^{-2x}$

(D) $c_1e^{2x} + c_2e^{-2x} + \frac{1}{x^2}$

46. Let $f(x, y) = x^5y^2\tan^{-1}\left(\frac{y}{x}\right)$. Then $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ is equal to

(A) $2f$

(B) $3f$

(C) $5f$

(D) $7f$

47. The integrating factor of the differential equation

$$\frac{dy}{dx} + P(x)y = Q(x)$$

is

(A) $e^{\int P(x)dx}$

(B) $e^{-\int P(x)dx}$

(C) $\int P(x)\,dx$

(D) $\int Q(x)\,dx$

48. If $y = \sin^{-1} x$, then

(A) $(1-x^2)\,y_{n+2} + (2n+1)x\,y_{n+1} - n^2 y_n = 0$

(B) $(1-x^2)\,y_{n+2} + (2n+1)\,y_{n+1} + n^2 y_n = 0$

(C) $(1-x^2)\,y_{n+2} - (2n+1)x\,y_{n+1} - n^2 y_n = 0$

(D) $(1-x^2)\,y_{n+2} - (2n+1)\,y_{n+1} + y_n = 0$

49. The value of $\left(\frac{\Delta^2}{E}\right)x^3$ is

(A) $4x^2 - 6x + 2$

(B) $6x$

(C) $\sqrt{2}x^3 + 9x^{-3} + 4$

(D) $x^3 + 3x^2 - 3x + 15$

50. If $u_0 = 3$, $u_1 = 12$, $u_2 = 81$, $u_3 = 200$, $u_4 = 100$ and $u_5 = 8$, then the value of $\Delta^5 u_0$ is

(A) 1362

(B) 855

(C) 755

(D) 655

51. A second-degree polynomial passes through (0, 1), (1, 3), (2, 7) and (3, 13). Then the polynomial is

(A) $x^2 + x + 1$ (B) $x^2 + 3x - 2$

(C) $x^2 - 5$ (D) $x^2 + 7x$

52. Let $I = \int_a^b y dx$, where $y = f(x)$. Consider the following two statements :

p : $y = f(x)$ is a polynomial of first degree in x for the trapezoidal rule.

q : $y = f(x)$ is a polynomial of third degree in x for Simpson's $\frac{3}{8}$th rule.

Choose the correct answer.

(A) p is true, q is true

(B) p is false, q is false

(C) p is true, q is false

(D) p is false, q is true

53. The reduction formula for $\int x^n e^{ax} dx$, n being a positive integer, is

(A) $I_n = \frac{x^{n-1}e^{ax}}{a} - \frac{n}{a} I_{n-1}$

(B) $I_n = \frac{x^{n-1}e^{ax}}{a} + \frac{n}{a} I_{n-1}$

(C) $I_n = \frac{x^n e^{ax}}{a} + \frac{n}{a} I_{n-1}$

(D) $I_n = \frac{x^n e^{ax}}{a} - \frac{n}{a} I_{n-1}$

54. The surface area of a sphere generated by the circle $x^2 + y^2 = a^2$ about the x-axis is

(A) πa^2

(B) $4\pi a^2$

(C) $2\pi a^2$

(D) $\frac{3}{4}\pi a^2$

55. The area of the centroid $r = a(1 - \cos\theta)$ is

(A) $\frac{1}{2}\pi a^2$

(B) $3\pi a^2$

(C) $\frac{3}{2}\pi a^2$

(D) $2\sqrt{\pi} a^2$

56. The area of the region enclosed by the curves $y(x^2 + 2) = 3x$ and $4y = x^2$ is given by

(A) $\int_{x=0}^{2} \int_{y=0}^{x^2/4} dx\, dy$

(B) $\int_{x=0}^{2} \int_{y=0}^{3x/(x^2+2)} dx\, dy$

(C) $\int_{x=0}^{1} \int_{y=x^2/4}^{3x/(x^2+2)} dx\, dy$

(D) $\int_{x=0}^{2} \int_{y=x^2/4}^{3x/(x^2+2)} dx\, dy$

57. The value of $\iint_R x^3 y\,dx\,dy$ over $R: \{0 \le x \le 1, 0 \le y \le 2\}$ is

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{1}{8}$

(D) $\frac{1}{16}$

58. The line which can be drawn from the point (2, – 1, 3) to intersect the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

and

$$\frac{x-4}{4} = \frac{y}{5} = \frac{z+3}{3}$$

at right angle is

(A) $\frac{x-2}{11} = \frac{y+1}{-10} = \frac{z-3}{2}$

(B) $\frac{x-2}{3} = \frac{y+1}{-4} = \frac{z-3}{5}$

(C) $\frac{x-2}{11} = \frac{y+1}{10} = \frac{z-3}{-8}$

(D) $\frac{x}{3} = \frac{y+1}{5} = \frac{z-3}{-7}$

59. Find the equation of the plane passing through the three points (2, 2, –1), (3, 4, 2) and (7, 0, 6).

(A) $5x + 2y - 3z + 17 = 0$

(B) $5x + 2y - 3z - 17 = 0$

(C) $2x + 5y - 17z + 3 = 0$

(D) $2x + 5y - 17z - 3 = 0$

60. The angle between the planes

$$2x - y + 3z + 7 = 0$$

and $x - 2y - 3z + 8 = 0$

is

(A) $\cos^{-1}\left(\frac{5}{14}\right)$

(B) $\pi / 3$

(C) $\cos^{-1}\left(\frac{-5}{14}\right)$

(D) $\cos^{-1}\left(\frac{-5}{7}\right)$

61. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by

$$T(x, y, z) = (x + y,\ y + z,\ z + x)$$

for all $(x, y, z) \in \mathbb{R}^3$. Then

(A) rank $(T) = 0$, nullity $(T) = 0$

(B) rank $(T) = 2$, nullity $(T) = 1$

(C) rank $(T) = 3$, nullity $(T) = 0$

(D) rank $(T) = 1$, nullity $(T) = 2$

62. The value of $\lim_{n \to \infty} n^{\frac{1}{n}}$ is

(A) greater than 1

(B) less than 1

(C) equal to 0

(D) equal to 1

63. The value of

$$\lim_{(x,\ y) \to (2,\ -2)} \frac{\sqrt{x-y} - 2}{x - y - 4}$$

is

(A) 0 (B) $\frac{1}{4}$

(C) $\frac{1}{3}$ (D) $\frac{1}{2}$

64. Consider the sequence $\{u_n\} = \{n^{(-1)^n n}\}$, where $n \geq 1$. Then

(A) $\overline{\lim}\, u_n = \infty$, $\underline{\lim}\, u_n = -1$

(B) $\overline{\lim}\, u_n = 1$, $\underline{\lim}\, u_n = -1$

(C) $\overline{\lim}\, u_n = \infty$, $\underline{\lim}\, u_n = 0$

(D) $\overline{\lim}\, u_n = \infty$, $\underline{\lim}\, u_n = -\infty$

65. The generating factor for the sequence $1, a, a^2, \ldots$, where a is a fixed constant, is

(A) $(1+a)^{-1} x$

(B) $(1-ax)^{-1}$

(C) $(1-a)^{-1} x$

(D) $(1+ax)^{-1}$

66. The sequence of functions $\{f_n\}_n$, $n \in \mathbb{N}$, where $f_n(x) = x^n$, $0 \leq x \leq 1$, is uniformly convergent on

(A) $[0, 1]$

(B) $(0, 1]$

(C) $(0, 1)$

(D) $[0, k]$ for $0 < k < 1$

67. The area of a triangle is 35 sq. units with vertices $(2, -6)$, $(5, 4)$ and $(k, 4)$. Then the value(s) of k is/are

(A) 12

(B) -2

(C) $-12, -2$

(D) $-2, 12$

68. The area of a rectangle having vertices A, B, C, D with position vectors $-\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}$, $\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}$, $\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$ and $-\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$ respectively, is

(A) 2

(B) 1

(C) 4

(D) $\frac{1}{4}$

69. The eccentricity of the ellipse $2x^2 + 3y^2 - 4x + 5y + 4 = 0$ is

(A) $\frac{1}{3}$

(B) $\frac{1}{\sqrt{3}}$

(C) 3

(D) $\sqrt{3}$

70. Which of the following conditions does not ensure the convergence of a real sequence (a_n)?

(A) $|a_n - a_{n+1}| \to 0$ as $n \to \infty$

(B) $\sum_{n=1}^{\infty} |a_n - a_{n+1}|$ is convergent

(C) $\sum_{n=1}^{\infty} n a_n$ is convergent

(D) The sequences (a_{2n+1}), (a_{2n+2}) and (a_{3n}) are convergent

71. Let $T:\mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T(1, 2) = (2, 3)$, $T(0, 1) = (1, 4)$. Then $T(5, 6)$ is

(A) $(6, -1)$

(B) $(-6, 1)$

(C) $(-1, 6)$

(D) $(1, -6)$

72. Let G be an Abelian group of order 10 and $S = \{g \in G : g^{-1} = g\}$. Then the number of non-identity elements in S is

(A) 5

(B) 2

(C) 1

(D) 0

73. Let $f:\mathbb{R} \to \mathbb{R}$ be a continuous function. If $\int_0^x f(2t)\,dt = \frac{x}{\pi}\sin(\pi x)$ for all $x \in \mathbb{R}$, then $f(2)$ is equal to

(A) -1

(B) 0

(C) 1

(D) 2

74. Let V be a vector space and let $L(S)$ denote the set of all linear combinations of members of S. Then which of the following is incorrect?

(A) $L(S)$ is a subspace of V

(B) $A \subset B \Rightarrow L(A)$ is a subspace of $L(B)$

(C) S is a subspace of V if and only if $L(S) = S$

(D) $A \neq B \Rightarrow L(A) \neq L(B)$

75. The order of $(8, 4, 10)$ in the group $Z_{12} \times Z_{60} \times Z_{24}$ is

(A) 6

(B) 12

(C) 60

(D) 360

76. The number of elements in the cyclic subgroup of Z_{30} generated by 25 is

(A) 3

(B) 4

(C) 5

(D) 6

77. The solutions of the equation $x^2 - 5x + 6 = 0$ in Z_{12} are

(A) 2, 3

(B) 2, 3, 6, 11

(C) 6, 11

(D) 3, 6, 9, 11

78. Choose the correct statement from the following.

(A) $\mathbb{Z}$ is not an integral domain.

(B) $\mathbb{Z}$ is a field.

(C) $\mathbb{Z}_2$ is not an integral domain.

(D) $M_2(\mathbb{Z}_2)$ has no divisor of zero.

79. Let N be a normal subgroup of a group G. Which one of the following is true?

(A) If G is an infinite group, then G/N is an infinite group

(B) If G is a non-Abelian group, then G/N is a non-Abelian group

(C) If G is a cyclic group, then G/N is an Abelian group

(D) If G is an Abelian group, then G/N is a cyclic group

80. Let R be the ring of all functions from $\mathbb{R}$ to $\mathbb{R}$ under the pointwise addition and multiplication. Let $I = \{f : \mathbb{R} \to \mathbb{R} :$ f is a bounded function$\}$ and $J = \{f : \mathbb{R} \to \mathbb{R} : f(3) = 0\}$. Choose the correct statement from the following.

(A) J is an ideal of R but I is not an ideal of R.

(B) I is an ideal of R but J is not an ideal of R.

(C) Both I and J are ideals of R.

(D) Neither I nor J is an ideal of R.

81. Let R be the ring of all 2×2 matrices with integer entries. Which of the following subsets of R is an integral domain?

(A) $\left\{ \begin{pmatrix} 0 & x \\ y & 0 \end{pmatrix} : x, y \in \mathbb{Z} \right\}$

(B) $\left\{ \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} : x, y \in \mathbb{Z} \right\}$

(C) $\left\{ \begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix} : x \in \mathbb{Z} \right\}$

(D) $\left\{ \begin{pmatrix} x & y \\ y & z \end{pmatrix} : x, y, z \in \mathbb{Z} \right\}$

82. In the group $\{1, 2, 3, \ldots, 16\}$ under the operation of multiplication modulo 17, the order of the element 3 is

(A) 4

(B) 8

(C) 12

(D) 16

83. If $\vec{u}(t) = u_1(t)\hat{i} + u_2(t)\hat{j} + u_3(t)\hat{k}$ is a unit vector and $\left|\frac{d\vec{u}}{dt}\right| \neq 0$, then the angle between $\vec{u}(t)$ and $\frac{d\vec{u}}{dt}$ is

(A) 0

(B) $\pi/2$

(C) $\pi/3$

(D) $\pi/4$

84. If a graph of five vertices has degrees of vertices 1, 4, 3, 3, 3 respectively, then its number of edges is

(A) 7

(B) 8

(C) 9

(D) 10

85. Let $G = (V, E)$ be a simple graph, where $|V| = \alpha$ and $|E| = \beta$. If G' is the complement of graph G, then how many edges does G' have?

(A) $\alpha - \beta$

(B) $\alpha + \beta$

(C) $\frac{\alpha(\alpha-1)}{2} - \beta$

(D) $\frac{\alpha(\alpha-1)}{2} + \beta$

86. An elliptical lamina is such that when it swings about one latus rectum as a horizontal axis, the other latus rectum passes through the centre of oscillation. The eccentricity is

(A) 4/7

(B) 2/3

(C) 1/2

(D) 2

87. If p and q are distinct primes such that $a^p \equiv a \pmod p$ and $a^q \equiv a \pmod q$, then

(A) $a^{pq} \equiv q \pmod{pq}$

(B) $a^{pq} \equiv p \pmod{pq}$

(C) $a^{pq} \equiv 0 \pmod p$

(D) $a^{pq} \equiv a \pmod{pq}$

88. How many edges must a planar graph have if it has 7 regions and 5 vertices?

(A) 8

(B) 10

(C) 12

(D) 14

89. The asymptotes to the curve $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$ are

(A) $y = \pm a$

(B) $y = \pm b$

(C) $x = \pm b$

(D) $x = \pm a$

90. Which of the following relations in the set of natural numbers is not reflexive?

(A) $x \leq y$

(B) x divides y

(C) x and y are relatively prime

(D) $x - y$ is divisible by 2

91. The general solution of the differential equation $(D^2 - 3D + 2)y = e^x$ is

(A) $y = c_1e^x + c_2e^{2x} - xe^x$

(B) $y = c_1 \cos x + c_2 \sin x - xe^x$

(C) $y = c_1e^x + c_2e^{2x} + xe^x$

(D) $y = c_1 \cos x + c_2 \sin x + xe^x$

92. If $I_n = \frac{d^n}{dx^n}(x^n \log x)$, then

(A) $I_n = nI_{n-1} + \underline{|n}$

(B) $I_n = nI_{n-1} + \underline{|n-1}$

(C) $I_n = nI_{n-1}$

(D) $I_n = nI_{n-1} + \underline{|n+1}$

93. The radius of curvature at the pole for the curve $r = a \sin n\theta$ is

(A) $\frac{1}{2} a(n+1)$

(B) $\frac{1}{2} a(n-1)$

(C) $\frac{1}{2} a\underline{|n}$

(D) $\frac{1}{2} a \cdot n$

94. The singular solution of the equation $y = px + \frac{a}{p}$, where $p = \frac{dy}{dx}$, is

(A) $y = 2ax + c$

(B) $y^3 = x^2$

(C) $y^2 = 4ax$

(D) $y^2 = ax + 2$

95. If $\vec{F} = (x^2 + y^3)\hat{i} + (x^3 - y^2)\hat{j}$, then the value of $\int_C \vec{F} \cdot d\vec{r}$ following the path $y^2 = x$ joining (0, 0) to (1, 1) is

(A) 19/35

(B) 128/331

(C) 5/9

(D) 2/3

96. A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$, where t is the time. The component of its velocity at time $t = 1$ in the direction $\hat{i} + \hat{j} + 3\hat{k}$ is

(A) $3\sqrt{11}$ (B) $\sqrt{15}$

(C) $\sqrt{13}$ (D) $\sqrt{11}$

97. The value of $\frac{1}{1 \times 3} + \frac{1}{5 \times 7} + \frac{1}{9 \times 11} + \dots$ is

(A) $\pi/8$ (B) $\pi/4$

(C) $\pi/2$ (D) $3\pi/4$

98. The remainder, when

$$\underline{|1} + \underline{|2} + \underline{|3} + \dots + \underline{|50}$$

is divided by 15, is

(A) 1 (B) 3

(C) 6 (D) 9

99. How many generators are there in the cyclic group of order 10?

(A) 1 (B) 2

(C) 3 (D) 4

100. The general solution of the differential equation

$$\frac{d^2y}{dx^2} - y = 2 + 3x$$

is

(A) $y = c_1e^x + c_2e^{2x} - 2 - 3x$

(B) $y = c_1e^x + c_2e^{-x} + 2 + 3x$

(C) $y = c_1e^x + c_2e^{-x} - 2 - 3x$

(D) $y = c_1e^x + c_2e^{-2x} + 2 - 3x$